Intermittent, Domain-Structured Coherence in the Pion Interferometry of Relativistic Nuclear Collisions

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Abstract

We investigate a domain-structured source in the pion interferometry of relativistic nuclear collisions. The source emits coherent pions intermittently with the background of chaotic pions. The coherent pions examined are either of a general nature or of disoriented chiral condensate. Two- and three-pion correlations for the source are shown to agree well with the recent NA44 experimental data.

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Two-pion interferometry has been used in relativistic nuclear collisions to extract information regarding the size and shape of the pion-emitting source formed during the collision [1]. Pion interferometry based on the Hanbury-Brown-Twiss (HBT) effect is not limited to that of two pions, but can also be of multi-pions, as in the case of three-pion interferometry. Though multi-pion interferometry is unavoidably complicated and obviously more difficult experimentally, it is expected to yield possibly new information that two-pion interferometry cannot provide [2,3].

Recently, the NA44 collaboration at CERN has reported a puzzling result from the first three-pion interferometry experiment [4]. Both the two- and the three-pion correlations are weak, with the three-pion correlation nearly vanishing. The strength of the two- and three-pion correlations is measured in terms of the chaoticity λ and the weight factor ω [2,3,5], the NA44 experiment yielding $\lambda = 0.4 - 0.5$ and $\omega = 0.20 \pm 0.02 \pm 0.19$.

When the source is completely chaotic, λ and ω are unity, while λ and ω vanish for a fully coherent source [7]. Over the years, it has been known [6] that a source consisting of two components, one coherent and the other chaotic, a so-called partially coherent source, gives $\lambda < 1$. Such a source would not yield, however, the NA44 values of λ and ω simultaneously. In Fig. 1, we illustrate a comparison of the partially coherent model [2] with the data. While $\lambda = 0.4 - 0.5$ corresponds to the source being about 70 % coherent, the weight factor of 0.20 $\pm$ 0.19 implies that it is nearly 100% coherent.

In this letter we examine a more complicated, believed to be more realistic, structure of the pion source that intermittently emits coherent pions from several regions with the background of chaotic pion emission. We also examine the consequences in pion interferometry when such coherent pions are from a domain structure of disoriented chiral condensate (DCC). We find that both satisfy the NA44 data.

For the pions emitted from several coherent regions with a chaotic background, we write the source current consisting of multi-components as [6,5]

$$J^i(x) = \sum_{n=1}^{N} j^i(x - X_n) e^{-i\theta_n} + J^i_{cha}(x), \tag{1}$$

where $j(x)$ and $J_{cha}(x)$ are coherent and chaotic source currents, respectively. The superscript i stands for the charge state of the pions, such as $+$, $-$, and 0. The n-th coherent domain is located at X_n, and is distributed according to $\rho(X_n)$ (appearing below), normalized as $\int \rho(x) d^4x = 1$. θ_n is a random phase, uniformly distributed over the range $[0, 2\pi)$. In order to describe an intermittent pion emission from a domain-structured source, *the coherent sources are taken to obey the Poisson distribution*

$$\gamma_N(\alpha) = \frac{\alpha^N}{N!} e^{-\alpha}. \tag{2}$$

We apply a generating functional method for these c-number source currents [8], defined (slightly differently from [8]) as

$$G[z_i^*(\mathbf{p}), z_i(\mathbf{p})] = \left\langle \exp\left\{ \sum_i \int \frac{d^3\mathbf{p}}{\sqrt{(2\pi)^3 \cdot 2p^0}} \left(z_i^*(\mathbf{p}) J^{i^*}(p) + z_i(\mathbf{p}) J^i(p) \right) \right\} \right\rangle_J , \tag{3}$$

where p^0 is on-shell. By taking functional derivatives of Eq. (3), one-, two- and three-pion spectra are then constructed as

$$W_1^i(p_1) = \langle |J^i(p_1)|^2 \rangle_J, \tag{4}$$
$$W_2^i(p_1,p_2) = \langle |J^i(p_1)|^2 |J^i(p_2)|^2 \rangle_J, \tag{5}$$
$$W_3^i(p_1,p_2,p_3) = \langle |J^i(p_1)|^2 |J^i(p_2)|^2 |J^i(p_3)|^2 \rangle_J, \tag{6}$$

where $J^i(p)$ is the Fourier transform of $J^i(x)$. The normalized two- and three-correlation functions are then expressed in terms of W's as

$$C_2^i(p_1,p_2) = \frac{\langle n_{\pi_i}\rangle^2}{\langle n_{\pi_i}(n_{\pi_i}-1)\rangle} \frac{W_2^i(p_1,p_2)}{W_1^i(p_1)W_1^i(p_2)}, \tag{7}$$
$$C_3^i(p_1,p_2,p_3) = \frac{\langle n_{\pi_i}\rangle^3}{\langle n_{\pi_i}(n_{\pi_i}-1)(n_{\pi_i}-2)\rangle} \frac{W_3^i(p_1,p_2,p_3)}{W_1^i(p_1)W_1^i(p_2)W_1^i(p_3)}, \tag{8}$$

where n_{π_i} is the number operator of the pions emitted.

$\langle \mathcal{O} \rangle_J$ is the statistical average of $\mathcal{O}$ about the fluctuation of $J_i(p)$.

$$\langle \mathcal{O} \rangle_J = \sum_{N=1}^{\infty} \gamma_N(\alpha) \int \left(\prod_{i=+,-,0} \mathcal{P}_i[J_{cha}^{i*}(p), J_{cha}^i(p)] \mathcal{D}J_{cha}^i(p) \mathcal{D}J_{cha}^{i*}(p) \right)$$
$$\times \int \left(\prod_{n=1}^{N} d^4X_N \rho(X_N) \right) \int_0^{2\pi} \left(\prod_{n=1}^{N} \frac{d\theta_n}{2\pi} \right) \mathcal{O}. \tag{9}$$

$\mathcal{P}_i[J_{cha}^{i*}(p), J_{cha}^i(p)]$ is a distribution functional of $J_{cha}^i(p)$, and assumed to have a Gaussian form, as in Ref. [8], so that the higher-order moment of $J_{cha}^i(p)$ is represented by the second-order moment, for example,

$$\langle J_{cha}^{i*}(p_1) J_{cha}^{i*}(p_2) J_{cha}^i(p_1) J_{cha}^i(p_2) \rangle_J =$$
$$\langle J_{cha}^{i*}(p_1) J_{cha}^i(p_1) \rangle_J \langle J_{cha}^{i*}(p_2) J_{cha}^i(p_2) \rangle_J + \langle J_{cha}^{i*}(p_1) J_{cha}^i(p_2) \rangle_J \langle J_{cha}^{i*}(p_2) J_{cha}^i(p_1) \rangle_J. \tag{10}$$

The chaoticity and the weight factor are defined as

$$\lambda_i = C_2^i(p,p) - 1, \tag{11}$$
$$\omega_i = \frac{C_3^i(p,p,p) - 3\lambda_i - 1}{2\sqrt{\lambda_i^3}}. \tag{12}$$

Usually, λ_i and ω_i are independent of the charge state, i, because of charge symmetry. We will not show i explicitly until we discuss the case of DCC. After some algebra [5], we obtain the chaoticity for the current, Eq. (1),

$$\lambda = \frac{\alpha}{\alpha + (1-\epsilon)^2}, \tag{13}$$

where α $(= \langle N \rangle)$ is the mean number of the coherent sources, and ϵ is the ratio of the number of pions emitted from the chaotic source and the total number of pions. When $\alpha \to \infty$ or $\epsilon \to 1$, Eq. (8) gives $\lambda \to 1$. This condition corresponds to a totally chaotic

source, in accordance with the description of a chaotic source given in [6], an infinite number of randomly distributed coherent sources. The weight factor is obtained, again after some algebra [5], as

$$\omega = \frac{2\alpha^2 + 8\alpha(1-\epsilon)^2 + 3(1-\epsilon)^3(1-2\epsilon)}{2\left(\alpha^2 + 3\alpha(1-\epsilon)^2 + (1-\epsilon)^3\right)} \sqrt{\frac{\alpha + (1-\epsilon)^2}{\alpha}}. \tag{14}$$

Note that $\omega \to 1$ as $\alpha \to \infty$ or $\epsilon \to 1$.

Figure 2 illustrates λ and ω when the parameters α and ϵ are varied. The best fit to the NA44 data [4], $\lambda = 0.45$ and $\omega = 0.20$, corresponds to $\alpha = 0.13$ and $\epsilon = 0.60$. This implies that one or two out of ten events contain coherent sources, the others being from totally chaotic sources, but about 40% of the emitted pions must come from the coherent sources. That is, very large coherent sources are sometimes created. Note that each of the coherent sources produces, on average, about five times more pions than the chaotic source.

We expect that the coherent sources are generated in association with a phase transition of quantum chromodynamics. Depending on specific dynamics involved in the generation, the coherent sources would have more complicated features and the preceding treatment would require some modification. In the following, as an example, we take a possible phenomenon of disoriented chiral condensate (DCC) [9–11].

The source current is now modified as [12]

$$J^i(p) = \sum_{n=1}^{N} j^i(p) e^{ip\cdot X_n - i\theta_n} n_n^i + J_g^i(p), \tag{15}$$

so as to incorporate the proposed consequence of DCC, the chiral order-parameter directed in the isospin space, differently from that in the true vacuum. Here, $\mathbf{n}_n$ is a unit-vector in the isospin space, describing the direction of condensate of the n-th domain. The statistical average $\langle \mathcal{O} \rangle_J$ of Eq. (9) is also modified to include

$$\int \left(\prod_{n=1}^{N} \frac{d^3\mathbf{n}_n}{4\pi} \delta(|\mathbf{n}_n| - 1) \right). \tag{16}$$

After lengthy algebra [12], we obtain the chaoticity and weight factor for the positive pions as

$$\lambda_+ = \frac{\alpha}{\alpha + \frac{6}{5}(1-\epsilon)^2}, \tag{17}$$

$$\omega_+ = \frac{\alpha^2 + \frac{6}{5}\alpha(1-\epsilon)^2 + \frac{27}{175}(1-\epsilon)^3(13-28\epsilon)}{\alpha^2 + \frac{18}{5}\alpha(1-\epsilon)^2 + \frac{54}{35}(1-\epsilon)^3} \sqrt{\frac{\alpha + \frac{6}{5}(1-\epsilon)^2}{\alpha}}. \tag{18}$$

For the neutral pions, we obtain

$$\lambda_0 = \frac{\alpha}{\alpha + \frac{9}{5}(1-\epsilon)^2}, \tag{19}$$

$$\omega_0 = \frac{1}{2} \frac{2\alpha^2 + \frac{18}{5}\alpha(1-\epsilon)^2 + \frac{81}{175}(1-\epsilon)^3(17-42\epsilon)}{\alpha^2 + \frac{27}{5}\alpha(1-\epsilon)^2 + \frac{27}{7}(1-\epsilon)^3} \sqrt{\frac{\alpha + \frac{9}{5}(1-\epsilon)^3}{\alpha}}. \tag{20}$$

Figure 3 shows that the multiple DCC domain model successfully yields the chaoticity and weight factor in agreement with the data. The parameter values for the best fit are found to be $\alpha (= \langle N \rangle) = 0.18$ and $\epsilon = 0.57$. This implies that the mean number of the DCC domains is 0.18 and that the ratio of the chaotic pion number and the total pion number is 0.57. For these parameter values, the probability distribution of the ratio of the neutral pion number and the total pion number, f, is

$$\begin{aligned} P_{\alpha,\epsilon}(f) &= \sum_{N=0}^{\infty} \frac{\alpha^N}{N!} e^{-\alpha} \int \delta \left(f - \frac{\sum_n^N 3 f_n (1-\epsilon) + \epsilon\alpha}{3N(1-\epsilon) + 3\epsilon\alpha} \right) \prod_{n=1}^{N} \frac{df_n}{2\sqrt{f_n}} \\ &\approx 0.83 P_0(f) + 0.15 P_1(f). \end{aligned} \tag{21}$$

Here, $P_0(f)$ is equal to $\delta(f - 1/3)$ in our simple model, while $P_1(f)$ is approximately the inverse square root of f. Since $P_1(f)$ is suppressed by the factor of 0.15, $P_{\alpha,\epsilon}(f)$ of Eq. (21) is a distribution dominated by a sharp peak with a slow-varying $1/\sqrt{f}$-like background. In practice, the sharp peak can be replaced by a smoother function such as a binomial distribution peaking at $f = 1/3$.

We have thus shown that the NA44 data can be explained by the model of multiple DCC with a chaotic pion source. We have not demonstrated, however, that the NA44 data prove the appearance of DCC. Unfortunately, in order to prove this solely by interferometry, we require interferometry of the neutral pions. For the above parameter values, the chaoticity and weight factor for the neutral pions are $\lambda_0 = 0.35$ and $\omega_0 = -0.12$, respectively. If interferometry of the neutral pions could be carried out, these values should signal the observation of DCC.

Note that the pion interferometry of differently charged pions does not serve for identifying DCC because the chaotic part exhibits no HBT effect and the DCC part is taken to be coherent.

The preceding direct fit to the NA44 data may be, however, associated with a substantial systematic error. In interferometry experiments, the events are often selected by "minimum bias," so as to improve statistics. The selection results in combining the events of different multiplicities, and for more detailed analyses a refinement is needed, such as taking account of event-by-event fluctuations.

We should comment on other possible reasons for the small weight factor in the experiment. First, as noted in [13], the contamination resulting from other particles being regarded as pions could make the weight factor smaller. According to our estimate, however, the contaminant ratio would have to be as large as about 30%, to obtain an agreement with the data. We would not expect that the accuracy of particle identification be this bad. Second, the long-lived resonances could cause the chaoticity to appear smaller [14]. This could happen, but the weight factor would not be affected by the decay process of resonances since the process is chaotic. Thus, if this should occur, the chaoticity would be smaller than expected, but the weight factor would remain as the expected value, contradicting the NA44 data. Third, some complicated final-state interactions could induce the apparent result. While we have not investigated all final-state interactions, we note that effects of the major final-state interactions were removed in the extraction of the NA44 data [4]. It is hard for us to expect that some other effects might bring about the puzzling data.

In summary, we introduce a domain-structured coherent source with the background of

a chaotic source. The pions are emitted intermittently, as the number of the domains is different in each event. We investigate the chaoticity and the weight factor, as measures of two- and three-pion correlations from such sources. While these quantities do not agree with the data in the simple partially coherent source model, they do agree in the newly introduced, domain-structured coherent-source model. Furthermore, when the source is associated with DCC, the quantities also agree with the NA44 data.

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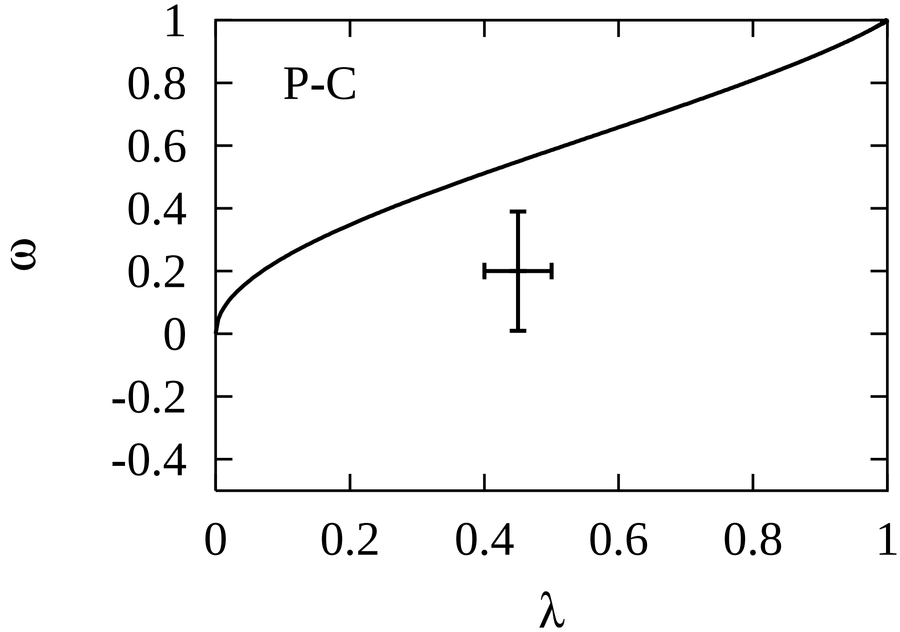

FIG. 1. Weight factor, ω, as a function of chaoticity, λ, for the model of a partially coherent source (P-C) [2] varying the relative coherency. The data is from the NA44 experiment [4].

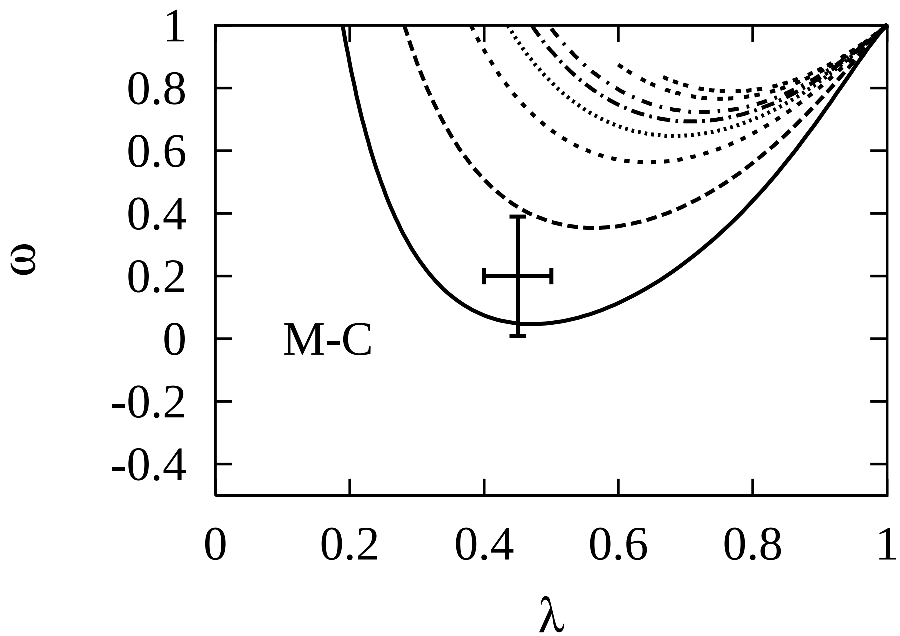

FIG. 2. Weight factor, ω, as a function of chaoticity, λ, for the positive pions, for the model of multiple coherent domains with one chaotic source (M-C), varying ϵ from 0 to 1. The lines from down to up correspond to the mean number of domains, $\langle N \rangle = 0.1$, 0.2, 0.4, 0.6, 0.8, 1.0, 1.5 and 2.0, respectively. The data is from the NA44 experiment [4].

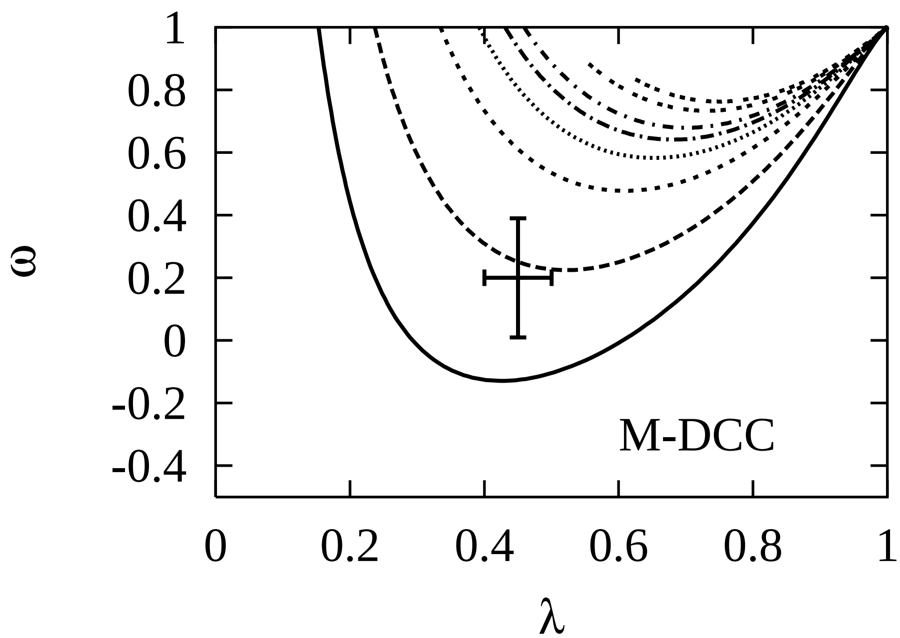

FIG. 3. Weight factor, ω, as a function of chaoticity, λ, for the positive pions, for the model of multiple DCC domains with one chaotic source (M-DCC), varying ϵ from 0 to 1. The lines from down to up correspond to the mean number of domains, $\langle N \rangle = 0.1$, 0.2, 0.4, 0.6, 0.8, 1.0, 1.5 and 2.0, respectively. The data is from the NA44 experiment [4].